

# Nagoya Termination Tool\*

Akihisa Yamada<sup>1</sup>, Keiichirou Kusakari<sup>2</sup>, and Toshiki Sakabe<sup>1</sup>

<sup>1</sup> Graduate School of Information Science, Nagoya University, Japan

<sup>2</sup> Faculty of Engineering, Gifu University, Japan

**Abstract.** This paper describes the implementation and techniques of the Nagoya Termination Tool, a termination prover for term rewrite systems. The main features of the tool are: the first implementation of the weighted path order which subsumes most of the existing reduction pairs, and the efficiency due to the strong cooperation with external SMT solvers. We present some new ideas that contribute to the efficiency and power of the tool.

## 1 Introduction

Proving termination of term rewrite systems (TRSs) has been an active field of research. In this paper, we describe the *Nagoya Termination Tool* (NaTT), a termination prover for TRS, which is available at

<http://www.trs.cm.is.nagoya-u.ac.jp/NaTT/>

NaTT is powerful and fast; its power comes from the novel implementation of the *weighted path order (WPO)* [24, 26] that subsumes most of the existing reduction pairs, and its efficiency comes from the strong cooperation with state-of-the-art *satisfiability modulo theory (SMT)* solvers. In principle, any solver that complies with the SMT-LIB Standard<sup>3</sup> version 2.0 can be incorporated as a back-end into NaTT.

In the next section, we recall the dependency pair framework that NaTT is based on, and present existing techniques that are implemented in NaTT. Section 3 describes the implementation of WPO and demonstrates how to obtain other existing techniques as instances of WPO. Some techniques on cooperating with SMT solvers are presented in Section 4. After giving some design details in Section 5, we assess the tool by its results in the *termination competition*<sup>4</sup> in Section 6. Then we conclude in Section 7. Due to page limit, some experimental results are found in the full version of this paper [25].

## 2 The Dependency Pair Framework

The overall procedure of NaTT is illustrated in Figure 1. NaTT is based on the

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<sup>3</sup> <http://www.smtlib.org/>

<sup>4</sup> [http://termination-portal.org/wiki/Termination\\_Competition](http://termination-portal.org/wiki/Termination_Competition)

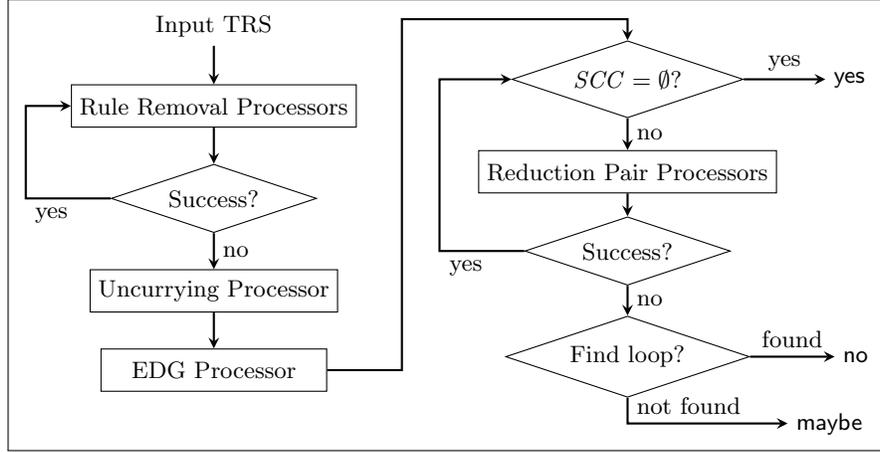


Fig. 1. Flowchart of NaTT

*dependency pair framework (DP framework)* [1,9,10], a very successful technique for proving termination of TRSs which is implemented in almost all the modern termination provers for TRSs. In the DP framework, dependencies between function calls defined in a TRS  $\mathcal{R}$  is expressed by the set  $\text{DP}(\mathcal{R})$  of *dependency pairs*. If a function  $f$  is defined by a rule

$$f(s_1, \dots, s_n) \rightarrow C[g(t_1, \dots, t_m)] \in \mathcal{R}$$

where  $g$  is also defined in  $\mathcal{R}$ , then this dependency is described by the following dependency pair:

$$f^\#(s_1, \dots, s_n) \rightarrow g^\#(t_1, \dots, t_m) \in \text{DP}(\mathcal{R})$$

The DP framework (dis)proves termination of  $\mathcal{R}$  by simplifying and decomposing *DP problems*  $\langle \mathcal{P}, \mathcal{R} \rangle$ , where initially  $\mathcal{P} = \text{DP}(\mathcal{R})$ . To this end, many *DP processors* have been proposed. NaTT implements the following DP processors:

**Dependency Graph Processor.** This processor decomposes a DP problem  $\langle \mathcal{P}, \mathcal{R} \rangle$  into  $\langle \mathcal{P}_1, \mathcal{R} \rangle \dots \langle \mathcal{P}_n, \mathcal{R} \rangle$  where  $\mathcal{P}_1, \dots, \mathcal{P}_n$  are the *strongly connected components (SCCs)* of the *dependency graph* [7,10]. Since the dependency graph is not computable in general, several approximations called *estimated dependency graphs (EDGs)* have been proposed. NaTT implements the EDG proposed in [8].

**Reduction Pair Processor.** This processor forms the core of NaTT. A *reduction pair* is a pair  $\langle \succsim, \succ \rangle$  of orders s.t.  $\succ$  is *compatible* with  $\succsim$  (i.e.,  $\succsim \cdot \succ \cdot \succsim \subseteq \succ$ ), both of  $\succsim$  and  $\succ$  are stable under substitution,  $\succsim$  is monotone and  $\succ$  is well-founded. From a DP problem  $\langle \mathcal{P}, \mathcal{R} \rangle$ , if all the involved rules are weakly decreasing (i.e.,  $\mathcal{P} \cup \mathcal{R} \subseteq \succsim$ ), strictly decreasing rules in  $\mathcal{P}$  (w.r.t.  $\succ$ ) can be removed. A great number of techniques for obtaining reduction pairs have been proposed so far. NaTT supports the following ones:

- Some *simplification orders* combined with *argument filters* [1]:
  - the *Knuth-Bendix order (KBO)* [14] and its variants including KBO with *status* [19], the *generalized KBO* [18] and the *transfinite KBO* [17, 21],
  - the *recursive path order* [3] and the *lexicographic path order (LPO)* [13],
- *polynomial interpretations (POLO)* [1, 16] and its variants, including certain forms<sup>5</sup> of *POLO* with negative constants [11] and *max-POLO* [6],
- the *matrix interpretation method* [4], and
- the *weighted path order (WPO)* [24, 26].

Note that all of the above mentioned reduction pairs are subsumed by WPO. That is, by implementing WPO we obtain the other reduction pairs for free. We discuss the implementation details in Section 3.

**Rule Removal Processor.** In the worst case, the size of dependency pairs is quadratic in the size of the input TRS  $\mathcal{R}$ . Hence it is preferable to reduce the size of  $\mathcal{R}$  before computing dependency pairs. To this end **NaTT** applies the *rule removal processor* [7]. If all rules in  $\mathcal{R}$  are weakly decreasing w.r.t. a *monotone* reduction pair, then the processor removes strictly decreasing rules from  $\mathcal{R}$ . The required monotonicity of a reduction pair is obtained by choosing appropriate parameters for the implementation of WPO described above.

**Uncurrying Processor.** Use of uncurrying for proving termination is proposed for *applicativ*e rewrite systems in [12]. The uncurrying implemented in **NaTT** is similar to the generalized version proposed in [20], in the sense that it does not assume *application symbols* to be binary. A symbol  $f$  is considered as an application symbol if all the following conditions hold:

- $f$  is defined and has positive arity,
- a subterm of the form  $f(x, \dots)$  does not occur in any left-hand-sides of  $\mathcal{R}$ ,
- a subterm of the form  $f(g(\dots), \dots)$  occurs in some right-hand-side of  $\mathcal{R}$ .

If such application symbols are found, then  $\mathcal{R}$  is uncurried w.r.t. the uncurrying TRS  $\mathcal{U}$  that consists of the following rules:<sup>6</sup>

$$f(f^l g(x_1, \dots, x_m), y_1, \dots, y_n) \rightarrow f^{l+1} g(x_1, \dots, x_m, y_1, \dots, y_n)$$

for every  $g \neq f$  and  $l$  less than the *applicativ*e arity<sup>7</sup> of  $g$ , where  $f^0 g$  denotes  $g$  and  $f^{l+1} g$  is a new function symbol of arity  $m + n$ .

### 3 The Weighted Path Order

As we mentioned in the introduction, **NaTT** implements only WPO for obtaining reduction pairs. WPO is parameterized by (1) a *weight algebra* which specifies

<sup>5</sup> Here, negative values are allowed only for the constant part.

<sup>6</sup> The notation is derived from the *freezing* technique [22].

<sup>7</sup> Applicative arities are taken so that  $\eta$ -saturation is not needed.

how weights are computed, (2) a *precedence* on function symbols, and (3) a *status function* which specifies how arguments are compared. In the following sections, we present some options which NaTT provides for specifying search spaces for these parameters.

### 3.1 Templates for Weight Algebras

One of the most important tasks in proving termination by WPO is finding an appropriate weight algebra. In order to reduce the task to an SMT problem, NaTT considers *template algebras* over integers. Currently the following template algebras are implemented:

- The algebra  $\mathcal{P}ol$  indicates that weights of terms are computed by a linear polynomial. Interpretations are in the following shape:

$$f_{\mathcal{P}ol}(x_1, \dots, x_n) = w_f + \sum_{i=1}^n c_{f,i} \cdot x_i \quad (1)$$

where the *template variables*  $w_f$  and  $c_{f,1}, \dots, c_{f,n}$  should be decided by an external SMT solver.

- The algebra  $\mathcal{M}ax$  indicates that weights are computed using the max operator. A symbol  $f$  with arity  $\geq 1$  is interpreted in the following shape:

$$f_{\mathcal{M}ax}(x_1, \dots, x_n) = \max_{i=1}^n (p_{f,i} + c_{f,i} \cdot x_i) \quad (2)$$

where  $p_{f,1}, \dots, p_{f,n}$  are template variables. For constant symbols, interpretations of the shape (1) are used. Since the operator max is not usually supported by SMT solvers, these interpretations are encoded as quantifier-free formulas using the technique presented in [24].

- The algebra  $\mathcal{M}Pol$  combines both forms of interpretations described above. Since it is inefficient to consider all combinations of these interpretations,  $\mathcal{M}Pol$  decides the shape of interpretations according to the following intuition: If a constraint such as  $f(x) > g(x, x)$  appears, then  $g$  is interpreted as  $g_{\mathcal{M}ax}$ , because the imposed constraint  $c_{f,1} \geq c_{g,1} \wedge c_{f,1} \geq c_{g,2}$  is easier than  $c_{f,1} \geq c_{g,1} + c_{g,2}$ , which would be imposed by the interpretation  $g_{\mathcal{P}ol}$ .

The template variables introduced above are partitioned into two groups: template variables  $w_f, p_{f,1}, \dots, p_{f,n}$  are grouped in the *constant part*, and template variables  $c_{f,1}, \dots, c_{f,n}$  are in the *coefficient part*. For efficiency, it is important to properly restrict the range of these variables.

### 3.2 Classes of Precedences

NaTT offers “quasi” and “strict” precedences, as well as an option to disable them (i.e., all symbols are considered to have the same precedence). For reduction pairs using precedences, we recommend quasi-precedences which are chosen by default, as the encoding follows the technique of [27] that naturally encodes quasi-precedences.

**Table 1.** Parameters for some monotone reduction pairs.

Technique	template	coefficient	constant	precedence	status
Linear POLO	$\mathcal{P}ol$	$\mathbb{Z}_+$	$\mathbb{N}$	no	empty
LPO	$\mathcal{M}ax$	$\{1\}$	$\{0\}$	yes	total
KBO <sup>8</sup>	$\mathcal{P}ol$	$\{1\}$	$\mathbb{N}$	yes	total
Transfinite KBO <sup>8</sup>	$\mathcal{P}ol$	$\mathbb{Z}_+$	$\mathbb{N}$	yes	total

### 3.3 Classes of Status Functions

NaTT offers three classes of *status functions*: “total”, “partial” and “empty” ones. The standard notions of *status functions* are total ones that were introduced to admit *permutation* of arguments when comparing them lexicographically from left to right (cf. [19]). Such a comparison appears in many well-known reduction pairs; famous examples are LPO and KBO. By combining the idea of argument filters, status functions have recently been generalized to *partial* ones, that do not only *permute* but may also drop some arguments [23]. A partial status is beneficial for KBO, and even more significant when combined with WPO [26]. The extreme case of a partial status is the “empty” status, that drops all arguments and so no comparison of arguments will be performed. This option corresponds to the nature of interpretation methods, e.g. POLO, if precedences are also disabled.

### 3.4 Obtaining Well-known Reduction Pairs

Although most of the existing reduction pairs are subsumed by WPO, some of them are still useful for improving efficiency, due to the restricted search space and simplified SMT encoding. We list parameters that correspond to some known reduction pairs in Tables 1 and 2. Note here that the effects of non-collapsing argument filters are simulated by allowing 0-coefficients in the weight algebra. Thus NaTT has a dedicated implementation only for *collapsing* argument filters, and implementations of usable rules for interpretation methods and path orders are smoothly unified.

## 4 Cooperation with SMT Solvers

NaTT is designed to work with any SMT-LIB 2.0 compliant solvers that support at least QF\_LIA logic, for which various efficient solvers exist.<sup>9</sup> NaTT extensively uses SMT encoding techniques for finding appropriate reduction pairs; the conditions of reduction pair processors are encoded into the following SMT constraint:

$$\bigwedge_{l \rightarrow r \in \mathcal{R}} \llbracket l \succ r \rrbracket \wedge \bigwedge_{s \rightarrow t \in \mathcal{P}} \llbracket s \succ t \rrbracket \wedge \bigvee_{s \rightarrow t \in \mathcal{P}} \llbracket s \succ t \rrbracket \quad (3)$$

<sup>8</sup> Further constraints for *admissibility* are imposed.

<sup>9</sup> Cf. the Satisfiability Modulo Theories Competition, <http://smtcomp.org/>.

**Table 2.** Parameters for some (non-monotone) reduction pairs.

Technique	template	coefficient	constant	precedence	status
Linear POLO	$\mathcal{P}ol$	$\mathbb{N}$	$\mathbb{N}$	no	empty
Max-POLO	$\mathcal{M}Pol$	$\mathbb{N}$	$\mathbb{Z}$	no	empty
LPO + argument filter	$\mathcal{M}ax$	$\{0, 1\}$	$\{0\}$	yes	total
KBO + argument filter	$\mathcal{P}ol$	$\{0, 1\}$	$\mathbb{N}$	yes	total
Matrix interpretations	$\mathcal{P}ol$	$\mathbb{N}^{d \times d}$	$\mathbb{N}^d$	no	empty
WPO( $\mathcal{M}Sum$ )	$\mathcal{M}Pol$	$\{0, 1\}$	$\mathbb{N}$	yes	partial

where each  $\llbracket l \succsim r \rrbracket$  is an SMT formula that represents the condition  $l \succsim r$ . In the remainder of this section, we present two techniques for handling such constraints that contribute to the efficiency of NaTT.

#### 4.1 Use of Interactive Features of SMT Solvers

In a typical run of termination verification, constraints of the form (3) are generated and solved many times, and each encoding sometimes involves thousands of lines of SMT queries with a number of template and auxiliary variables. Hence runtime spent for the SMT solver forms a large part of the overall runtime of the tool execution. NaTT tries to reduce the runtime by using *interactive* features of SMT solvers,<sup>10</sup> which are specified in SMT-LIB 2.0.

For each technique of reduction pairs, the encoded formula of the constraint  $\bigwedge_{l \rightarrow r \in \mathcal{R}} \llbracket l \succsim r \rrbracket$  need not be changed during a run, as far as  $\mathcal{R}$  is not modified.<sup>11</sup> Hence, when a reduction pair processor is applied for the first time, the back-end SMT solver is initialized according to the following pseudo-script:

```
(assert ( $\bigwedge_{l \rightarrow r \in \mathcal{R}} (u_{l \rightarrow r} \Rightarrow \llbracket l \succsim r \rrbracket)$ ))
(push)
```

where  $u_{l \rightarrow r}$  is a boolean variable denoting whether the rule  $l \rightarrow r$  is usable or not. When the processor is applied to an SCC  $\mathcal{P}$ , the following script is used:

```
(assert ( $\bigwedge_{s \rightarrow t \in \mathcal{P}} \llbracket s \succsim t \rrbracket \wedge \bigvee_{s \rightarrow t \in \mathcal{P}} \llbracket s \succ t \rrbracket$ ))
(check-sat)
```

Then, if a solution is found by the SMT solver, NaTT analyzes the solution using the `get-value` command. After this analysis, the command

```
(pop)
```

is issued to clear the constraints due to  $\mathcal{P}$  and go back to the context saved by the `(push)` command. In order to derive the best performance of the solver,

<sup>10</sup> NaTT is not the first tool to use the interactive features of SMT solvers; e.g., Boogie makes use of these features in its Houdini implementation [15].

<sup>11</sup> Although rules in  $\mathcal{R}$  may be removed by considering *usable rules*, the formula still need not be changed, since it can be simulated by negating a propositional variable that represents whether the rule is usable or not.

`(reset)`

is also issued in case sufficiently many rules become unusable (e.g., 1/3 of the rules in  $\mathcal{R}$ ) from  $\mathcal{P}$ . All these commands, `push`, `pop` and `reset` are expected to be available in SMT-LIB 2.0 compliant solvers.

## 4.2 Use of Linear Arithmetic

Note that expressions of the form (1) or (2) are nonlinear, due to the coefficients  $c_{f,1}, \dots, c_{f,n}$ . However, not many SMT solvers support *nonlinear* arithmetic, and even if they do, they are much less scalable than they are for linear arithmetic. Hence, we consider reducing the formulas to linear ones by restricting the range of  $c_{f,1}, \dots, c_{f,n}$  e.g. to  $\{0, 1\}$ . Although the idea is inspired by [2], NaTT uses a more straightforward reduction using `ite` (*if-then-else*) expressions. Each coefficient  $c_{f,i}$  is replaced by the expression `(ite  $b_{f,i}$  1 0)` where  $b_{f,i}$  is a propositional variable, and then multiplications are reduced according to the rule:

$$(* (ite\ e_1\ e_2\ e_3)\ e_4) \rightarrow (ite\ e_1\ (*\ e_2\ e_4)\ (*\ e_3\ e_4))$$

It is easy to see that this reduction terminates and linearizes expressions of the form (1) or (2). It is also possible to avoid an explosion of the size of formulas by introducing a auxiliary variable for the duplicated expression  $e_4$ .

*Example 1.* Consider the constraint  $f(f(a)) > b$  interpreted in the algebra  $\mathcal{Pol}$ , and suppose that the range of  $c_{f,1}$  is restricted to  $\{1, 2\}$ . The interpretation of the term  $f(f(a))$  is reduced as follows (written as S-expressions):

$$\begin{aligned} \llbracket f(f(a)) \rrbracket &= (+\ w_f\ (*\ (ite\ b_{f,1}\ 2\ 1)\ \llbracket f(a) \rrbracket)) \\ &\rightarrow (+\ w_f\ (ite\ b_{f,1}\ (*\ 2\ \llbracket f(a) \rrbracket)\ \llbracket f(a) \rrbracket)) \end{aligned}$$

Similarly, for  $f(a)$  we obtain

$$\llbracket f(a) \rrbracket \rightarrow (+\ w_f\ (ite\ b_{f,1}\ (*\ 2\ w_a)\ w_a))$$

Now, the constraint  $\llbracket f(f(a)) > b \rrbracket$  is expressed by the following script:

```
(define-fun v (+ w_f (ite b_{f,1} (* 2 w_a) w_a)))
(assert (> (+ w_f (ite b_{f,1} (* 2 v) v) w_b)))
```

In contrast to SAT encoding techniques [4–6], we do not have to care about the bit-width for the constant part and intermediate results. It is also possible to indicate that NaTT should keep formulas nonlinear, and solve them using SMT solvers that support QF\_NIA logic. Our experiments on TPDB<sup>12</sup> problems, however, suggests that use of nonlinear SMT solving is impractical for our purpose.

<sup>12</sup> The Termination Problem Data Base, <http://termination-portal.org/wiki/TPDB>.

## 5 Design

The source code of NaTT consists of about 6000 lines of code written in OCaml.<sup>13</sup> About 23% is consumed by interfacing SMT solvers, where some optimizations for encodings are also implemented. Another 17% is for parsing command-lines and TRS files. The most important part of the source code is the 40% devoted to the implementation of WPO, the unified reduction pair processor. Each of the other processors implemented consumes less than 3%. For computing SCCs, the third-party library `ocamlgraph`<sup>14</sup> is used.

### 5.1 Command Line Interface

The command line of NaTT has the following syntax:

```
./NaTT [FILE] [OPTION]... [PROCESSOR]...
```

To execute NaTT, an SMT-LIB 2.0 compliant solver must be installed. By default, z3 version 4.0 or later<sup>15</sup> is supposed to be installed in the path. Users can specify other solvers by the `--smt "COMMAND"` option, where the solver invoked by `COMMAND` should process SMT-LIB 2.0 scripts given on the standard input.

The TRS whose termination should be verified is read from either the specified `FILE` or the standard input.<sup>16</sup> Each `PROCESSOR` is either an order (e.g. `POLO`, `KBO`, `WPO`, *etc.*, possibly followed by options), or a name of other processors (`UNCURRY`, `EDG`, or `LOOP`). Orders preceding the `EDG` processor should be monotone reduction pairs and applied as rule removal processors before computing the dependency pairs. Orders following the `EDG` processor are applied as reduction pair processors to each SCC in the `EDG`. A list of available `OPTIONS` and `PROCESSORS` can be obtained via `NaTT --help`.

### 5.2 The Default Strategy

In case no `PROCESSOR` is specified, the following default strategy will be applied:

- As a rule removal processor, `POLO` with coefficients in  $\{1, 2\}$  and constants in  $\mathbb{N}$  is applied.
- Then the uncurrying processor is applied.
- The following reduction pair processors are applied (in this order):
  1. `POLO` with coefficients in  $\{0, 1\}$  and constants in  $\mathbb{N}$ ,
  2. algebra *Max* with coefficients in  $\{0, 1\}$  and constants in  $\mathbb{N}$ ,
  3. `LPO` with quasi-precedence, status and argument filter,
  4. algebra *MPol* with coefficients in  $\{0, 1\}$  and constants in  $\mathbb{Z}$ ,
  5. `WPO` with quasi-precedence, partial status, algebra *MPol*, coefficients in  $\{0, 1\}$  and constants in  $\mathbb{N}$ ,
  6. matrix interpretations with  $\{0, 1\}^{2 \times 2}$  matrices and  $\mathbb{N}^2$  vectors.
- If all the above processors fail, then a (naive) loop detection is performed.

<sup>13</sup> <http://caml.inria.fr/>

<sup>14</sup> <http://ocamlgraph.lri.fr/>

<sup>15</sup> <http://z3.codeplex.com/>

<sup>16</sup> The format is found at <https://www.lri.fr/~marche/tpdb/format.html>.

## 6 Assessment

Many tools have been developed for proving termination of TRSs, and the international termination competition has been held annually for a decade. NaTT participated in the *TRS Standard* category of the full-run 2013, where the other participants are versions of: AProVE,<sup>17</sup> TTT<sub>2</sub>,<sup>18</sup> MU-TERM,<sup>19</sup> and WANDA.<sup>20</sup> Using the default strategy described in Section 5.2, NaTT (dis)proves termination of 982 TRSs out of 1463 TRSs, and comes next to (the two versions of) AProVE, the constant champion of the category. It should be noticed that NaTT proved termination of 34 TRSs out of the 159 whose termination could not be proved by any other tool. NaTT is notably faster than the other competitors; it consumed only 21% of the time compared to AProVE, the second fastest. We expect that we can further improve efficiency by optimizing to multi-core architecture; currently, NaTT runs in almost single thread. NaTT also participated in the *SRS Standard* category. However, the result is not as good as it is for TRSs. This is due to the fact that the default strategy of Section 5.2 is designed only for non-unary signatures. It should be improved by choosing a strategy depending on the shape of input TRSs.

## 7 Conclusion

We described the implementation and techniques of the termination tool NaTT. The novel implementation of the weighted path order is described in detail, and some techniques for cooperating SMT solvers are presented. Together with these efforts, NaTT is one of the most efficient and strongest tools for proving termination of TRSs.

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<sup>17</sup> <http://aprove.informatik.rwth-aachen.de/>

<sup>18</sup> <http://cl-informatik.uibk.ac.at/software/ttt2/>

<sup>19</sup> <http://zenon.dsic.upv.es/muterm/>

<sup>20</sup> <http://wandahot.sourceforge.net/>

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